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On the 3-Local Subgroups of Conway's Group Co_1

ROBERT A. WILSON

Department of Pure Mathematics and Mathematical Statistics, Univeristy of Cambridge, 16 Mill Lane, Cambridge CB2 1SB, United Kingdom

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The maximal p-local subgroups of Conway's group Co_1 were determined by R. T. Curtis in [2], and the non-local maximal subgroups by the present author in [3]. Unfortunately there are some (mainly arithmetical) errors in the 3-local analysis which mean that the groups (vii) and (viii) of Theorem 3.1 of [2] do not exist. Thus (xxiii) and (xxiv) should be deleted from the Corollary in [3], and the 18th and 20th rows should be deleted from the list of maximal subgroups of Co_1 given in the ATLAS [1]. As the arguments used to establish such results are rather delicate, we repeat the whole proof here.

Let E be an elementary Abelian 3-group in Co_1. We begin by summarising the argument used in [2] to reduce to the case where E is an elementary Abelian group of order 9 all of whose non-trivial elements are of class $3C$. (Note: we use ATLAS notation—these are called B-elements in [2].)

First, in $C(3D) \cong 3 \times A_9$ a 3-element is of class $3D$ if and only if it is not in the A_9. Hence we may neglect $3D$-elements, as in Lemma 3.1 of [2].

Second, a $3B$-element is uniquely the product of two commuting $3A$-elements. Hence if E contains elements of class $3A$ or $3B$ we may assume, as in Lemma 3.2 of [2], that E is generated by $3A$-elements. It is not difficult to show in this case that $N(E)$ is contained in one of the groups (i), (iv), (v), or (vi) of Theorem 3.1 of [2]. (This is done on pages 424–425 of [2], and, essentially, also in Section 2.2 of [4].)

Third, if χ is the degree 24 character of $2Co_1$, then $\chi(3C) = -3$. Hence if E is $3C$-pure then it has order at most 9.

Now χ restricts to $C_{2Co_1}(3C) \cong 2 \times 3_+^{1+4}$: $\mathrm{Sp}_4(3)$ as the sum of an extension of each of the two faithful degree 9 characters of 3_+^{1+4}, together with the degree 6 irreducible character of $PSp_4(3)$. Thus since $\chi(3C) = \chi(9A) = -3$, and a $9A$-element cubes to a $3C$-element, it follows that any $3C^2$-group or $9A$-group maps to an element of class $3AB$ in the quotient $PSp_4(3)$ of $C(3C)$.

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Such an element corresponds to a transvection t of $Sp_4(3)$, so $C(t)\cong 2\times 3_+^{1+4}{:}\,2A_4$ has four orbits, of sizes 1, 2, 24, and 54, on the vectors of the natural module 3^4. Since the union of the first two orbits is $\mathrm{Im}(1-t)$, it follows that $C(t)$ has just three orbits, of sizes 1, 8, and 18, on the vectors of $3^4/\mathrm{Im}(1-t)$.

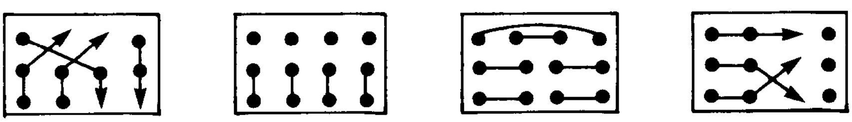
Now if t lifts to an element $\tilde{t}$ of $3^4{:}\,Sp_4(3)$, then $\tilde{t}$ is conjugate to its multiples by vectors in $\mathrm{Im}(1-t)$, so this means that there are just three classes of such elements $\tilde{t}$, with centralizers of orders $2^4\cdot 3^7$, $2\cdot 3^7$, and $2^3\cdot 3^5$, respectively.

These lift to three classes of groups of order 9 in $3_+^{1+4}{:}\,Sp_4(3)$. In the first case the centralizer order increases by a factor of 3, while in the other two cases it does not. From the character values, the first of these is a 3^2-group of type $3ABCC$. From the centralizer orders, the third contains $9A$-elements. Thus by elimination the second is a 3^2-group of pure $3C$-type. Its normalizer in $C_{Co_1}(3C)$ has order $2\cdot 3^8$, so its full normalizer in Co_1 has order $2^4\cdot 3^8$.

Finally, we show that $N(E)\leqslant 3^6{:}2M_{12}$. For a $3C$-element in the 3^6 corresponds to a Golay code word of weight 9, so we may take E to be generated by

$$\begin{array}{|cccc|}\hline + & + & + & 0\\ + & + & + & 0\\ + & + & + & 0\\ \hline\end{array} \qquad \text{and} \qquad \begin{array}{|cccc|}\hline + & 0 & - & -\\ 0 & - & + & +\\ 0 & - & + & +\\ \hline\end{array}$$

Then E is visibly normalized by the subgroup $S_3\times A_4$ of M_{12} generated by the elements



(see [1, pp. 31–32]), so by order considerations we have $N(E)\cong 3^6{:}\,(S_3\times 2A_4)\leqslant 3^6{:}\,2M_{12}$, as required.

References

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